Ratios

Unit Overview
In this unit you will study ratios, rates, proportions, and percents as you explore applications and use them to solve problems.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- benchmark

Math Terms
- ratio
- equivalent ratios
- rate
- dimensional analysis
- conversion factor
- unit rate
- unit price
- proportion
- percent
- average

ESSENTIAL QUESTIONS
Why is it important to understand calculations with ratios, rates, and percents?
Why are proportional relationships important in mathematics?

EMBEDDED ASSESSMENTS
These assessments, following activities 19 and 21, will give you an opportunity to demonstrate your ability to work with ratios, rates, and percents to solve mathematical and real-world problems involving proportional relationships.

Embedded Assessment 1:
Ratios and Rates p. 245

Embedded Assessment 2:
Understanding and Applying Percents p. 273
1. Label the scale on each number line as indicated.
   a. \[3 \quad 6 \quad 18\]
   b. 1 to 2

2. Identify each pair of fractions that are equal.
   a. \(\frac{2}{3}\) and \(\frac{4}{5}\)
   b. \(\frac{5}{8}\) and \(\frac{10}{16}\)
   c. \(\frac{3}{7}\) and \(\frac{7}{15}\)
   d. \(\frac{2}{5}\) and \(\frac{5}{10}\)
   e. \(\frac{3}{5}\) and \(\frac{9}{15}\)

3. Use division to find an equivalent decimal. Round quotients to the nearest hundredth.
   a. \(\frac{3}{8}\)
   b. \(\frac{6}{11}\)
   c. \(\frac{6}{9}\)
   d. \(\frac{5}{7}\)

4. Complete each of the following:
   a. 1 foot = _____ inches
   b. 1 yard = _____ inches
   c. 1 hour = _____ minutes
   d. 1 hour = _____ seconds
   e. 1 cup = _____ ounces
   f. 1 pound = _____ ounces
   g. 1 dime = _____ pennies

5. Place the following numbers in a Venn diagram to create a visual representation.
   A: Whole numbers less than 12
   B: Prime numbers less than 15

6. Find the value of each of the following.
   a. \(3.68 \div 4\)
   b. \(8.94 \div 6\)
   c. \(10.32 \div 8\)

7. a. Shade \(\frac{1}{3}\) of the figure.
   b. Shade \(\frac{2}{5}\) of the figure.

8. Solve each of the following for \(x\).
   a. \(7x = 21\)
   b. \(4x = 10\)
   c. \(1.2x = 24\)
   d. \(2.5x = 6\)
Understanding Ratios
All About Pets
Lesson 17-1 Understanding Ratios

Learning Targets:
- Understand the concept of a ratio and use ratio language.
- Represent ratios with concrete models, fractions, and decimals.
- Give examples of ratios as multiplicative comparisons of two quantities describing the same attribute.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Look for a Pattern

A **ratio** is a comparison of two quantities. Ratios can represent a comparison of part-to-part, part-to-whole, or whole-to-part. Ratios can be written as fractions, or using the word “to” or a colon.

**Example A**
Use the tags below. Find each of these ratios:

a. stars to bones
b. stars to total number of tags
c. total number of tags to bones

Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

**Solution:**
a. stars to bones
   - part-to-part: \[\frac{\text{number of stars}}{\text{number of bones}} = \frac{4}{8}; 4 \text{ to } 8, 4:8\]
b. stars to total number of tags
   - part-to-whole: \[\frac{\text{number of stars}}{\text{number of tags}} = \frac{4}{12}; 4 \text{ to } 12, 4:12\]
c. total number of tags to bones
   - whole-to-part: \[\frac{\text{number of tags}}{\text{number of bones}} = \frac{12}{8}; 12 \text{ to } 8, 12:8\]

**Try These A**
Use ratios to compare the pet toys shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

a. balls of yarn to mice
b. white balls of yarn to total number of toys
c. gray mice to white mice

Like fractions, ratios can sometimes be rewritten in lowest terms.
\[
\frac{4}{8} = \frac{1}{2}, 1 \text{ to } 2, \text{ or } 1:2
\]
\[
\frac{4}{12} = \frac{1}{3}, 1 \text{ to } 3, \text{ or } 1:3
\]
\[
\frac{12}{8} = \frac{3}{2}, 3 \text{ to } 2, \text{ or } 3:2
\]
A ratio is also a multiplicative comparison of two quantities. The ratio of circles to the total number of shapes below is \( \frac{2}{5} \).

This means \( \frac{2}{5} \) of all the shapes are circles and that for every 2 circles added, a total of 5 shapes will be added. Suppose a set of shapes with the pattern above includes 8 circles. You know that \( 2 \times 4 = 8 \), so multiply the number of shapes in the repeating part of the set (2 circles + 3 squares = 5 shapes) by 4 to find the total number of shapes when there are 8 circles: \( 5 \times 4 = 20 \) total shapes.

**Example B**

**Make sense of problems.** In January, for every 3 cats adopted, 4 dogs were adopted. A total of 16 dogs were adopted. How many cats were adopted?

**Step 1:** Write a ratio comparing the number of cats to the number of dogs adopted.

\[
\frac{\text{number of cats}}{\text{number of dogs}} = \frac{3}{4}
\]

The number of cats adopted is \( \frac{3}{4} \) times the number of dogs adopted.

**Step 2:** Multiply the ratio times the number needed to create an equivalent ratio showing 16 dogs.

\[
\frac{3}{4} \times \frac{4}{4} = \frac{12 \text{ cats}}{16 \text{ dogs}}
\]

**Solution:** 12 cats were adopted.

**Check:** Does the ratio of 12 cats to 16 dogs equal \( \frac{3}{4} \)?

\[
\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}
\]

**Try These B**

At the dog park on Monday, 2 dogs out of every 5 were terriers. A total of 20 dogs were at the park.

a. How many terriers were there? Explain how you got your answer.

b. The ratio of Irish terriers to the total number of terriers was 1:4. How many of the terriers were Irish terriers? Explain how you got your answer.
Lesson 17-1
Understanding Ratios

Check Your Understanding

1. For a given ratio, how many equivalent ratios can be written? Explain your reasoning.

2. How can you check to see if the ratio 1:2 is equivalent to another ratio?

3. Find as many whole-number ratios equal to 50:100 as you can, using division.

LESSON 17-1 PRACTICE

4. Use ratios to compare the dog bowls shown. Write each ratio three different ways. State whether the ratio is a part-to-part, part-to-whole, or whole-to-part.

a. white bowls to total number of bowls
b. black bowls to gray bowls
c. all bowls to bowls that are not gray

5. At the veterinarian’s office, 4 animals out of every 5 seen were cats. A total of 35 animals were seen.
   a. How many cats were seen?
   b. The ratio of male cats to all cats seen was 6:7. How many of the cats seen were males?

6. There are twelve rabbits in a cage. The ratio of white rabbits to all rabbits is 3:4. How many white rabbits are in the cage?

7. Make sense of problems. Each veterinarian has seen 40 animals today. Two out of every 5 animals Vet A has seen have been dogs. Three out of every 8 animals Vet B has seen have been dogs. Which vet saw more dogs today? Explain your reasoning.

8. Reason abstractly. The ratio of red collars to black collars sold at one store is 9 to 10. In one month 30 black collars are sold. Is 57 a reasonable number for the total number of red and black collars sold that month? Explain your reasoning.

9. There are 15 black mice in a cage. The ratio of all mice to black mice is 5:1. How many mice are in the cage?
Learning Targets:
• Make tables of equivalent ratios relating quantities.
• Use tables to compare ratios.
• Plot the pairs of values on the coordinate plane and describe the relationship.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Visualization, Create Representations, Identify a Subtask

Relationships that have equivalent ratios are called proportional relationships. All the columns in a ratio table show equivalent ratios.

Example A
Reason quantitatively. A recipe for a homemade dog treat calls for a mixture of 8 ounces of oats to 12 ounces of finely chopped liver. Complete the ratio table.

<table>
<thead>
<tr>
<th>Oats (oz)</th>
<th>Liver (oz)</th>
<th>(8 \div 2)</th>
<th>(8 \times 2)</th>
<th>(8 \times 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8 \div 4)</td>
<td>(8 \div 2)</td>
<td>(8 \div 4)</td>
<td>(8 \times 2)</td>
<td>(8 \times 10)</td>
</tr>
<tr>
<td>(8 \times 2)</td>
<td>(8 \times 2)</td>
<td>(8 \times 2)</td>
<td>(8 \times 10)</td>
<td>(8 \times 10)</td>
</tr>
<tr>
<td>(8 \times 10)</td>
<td>(8 \times 10)</td>
<td>(8 \times 10)</td>
<td>(8 \times 10)</td>
<td>(8 \times 10)</td>
</tr>
</tbody>
</table>

a. How many ounces of liver are needed with 16 oz of oats?
Solution: 24 oz of liver are needed with 16 oz of oats.

b. How many ounces of oats are needed with 120 oz of liver?
Solution: 80 oz of oats are needed with 120 oz of liver.

c. Use the table to name four ratios equivalent to \(\frac{8}{12}\).
Solution: The ratios \(\frac{2}{3}\), \(\frac{4}{6}\), \(\frac{16}{24}\), and \(\frac{80}{120}\) are equivalent to \(\frac{8}{12}\).

Try These A
a. In one recipe for dog biscuits, the ratio of cups of water to cups of flour used is 3:9. Complete the ratio table.

<table>
<thead>
<tr>
<th>Water (c)</th>
<th>3 (\div 3)</th>
<th>3 (\times 2)</th>
<th>3 (\times 4)</th>
<th>3 (\times 6)</th>
<th>3 (\times 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flour (c)</td>
<td>3 (\div 3)</td>
<td>3 (\times 2)</td>
<td>3 (\times 4)</td>
<td>3 (\times 6)</td>
<td>3 (\times 9)</td>
</tr>
<tr>
<td>(9 \div 3)</td>
<td>(9 \times 2)</td>
<td>(9 \times 4)</td>
<td>(9 \times 6)</td>
<td>(9 \times 9)</td>
<td>(9 \times 9)</td>
</tr>
</tbody>
</table>

b. How many cups of water are needed with 81 cups of flour?
c. How many cups of flour are needed with 12 cups of water?
d. Use the table to name five ratios equivalent to 3:9.
Lesson 17-2
Ratios in Proportional Relationships

A relationship is proportional if the graph of the relationship is a set of points through which a straight line can be drawn and the straight line passes through the point (0, 0).

Example B
At the animal food store, 20 dog biscuits cost $6. Is the relationship between the number of biscuits and the cost proportional?

Step 1: Make a ratio table.

<table>
<thead>
<tr>
<th>Number of Biscuits, x</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($), y</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

Step 2: Graph the relationship between the number of biscuits $x$ and the cost $y$. Plot the ordered pairs $(x, y)$ from the table: $(10, 3), (20, 6), (40, 12),$ and $(60, 18).

Solution: A line passes through all the points and through $(0, 0)$. This means that the relationship is proportional.

Try These B
Graph each relationship in the My Notes section to the right. Determine if the relationship is proportional or not proportional. Explain your reasoning.

a.

<table>
<thead>
<tr>
<th>Number of Hours, $x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($), $y$</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

b.

<table>
<thead>
<tr>
<th>Number of Hours, $x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($), $y$</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>27</td>
</tr>
</tbody>
</table>
Lesson 17-2
Ratios in Proportional Relationships

**Check Your Understanding**

1. How can you use a ratio table to find the value of $x$ in the ratio $x:20$ if the ratio is equivalent to $5:2$? Explain your reasoning.

2. Name two ways to determine if the $x$- and $y$-values in a table have a proportional relationship.

**LESSON 17-2 PRACTICE**

3. **Reason quantitatively.** The recipe for a homemade dog treat calls for a mixture of 2 eggs for every 8 cups of flour.
   a. Complete the ratio table.
      
      | Number of Eggs | 1 | 2 | 6 |
      |----------------|---|---|---|
      | Cups of Flour  | 8 | 40 | 64 |

   b. How many eggs are needed with 40 cups of flour?
   c. How many cups of flour are needed with 6 eggs?
   d. Use the table to name four ratios equivalent to $\frac{2}{8}$.
   e. Which ratio is equivalent to 2:8 in lowest terms?

4. **Model with mathematics.** For every 4 days of dog sitting Julie charges $20.
   a. Complete the table to find the amount Julie should charge for 1, 2, and 8 days of dog sitting.
      
      | Number of Days, $x$ | 1 | 2 | 4 | 8 |
      |---------------------|---|---|---|---|
      | Total Cost ($), $y$ | 20 |

   b. Graph the relationship between the number of days $x$ and the cost $y$.
   c. Is the relationship between the number of days and the cost proportional? Justify your answer.
   d. Use your graph to determine how much Julie should charge for 6 days of dog sitting.
   e. Is 4:20 equivalent to 10:50? Explain using the graph.

5. Are $\frac{2}{3}$ and $\frac{5}{6}$ equivalent ratios? Justify your answer.

6. Are $\frac{2}{7}$ and $\frac{6}{21}$ equivalent ratios? Justify your answer.

7. Are $\frac{2}{4}$ and $\frac{3}{6}$ equivalent ratios? Justify your answer.
ACTIVITY 17 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 17-1

1. Write a ratio in three different ways to represent the number of boys to the number of girls in the class.

<table>
<thead>
<tr>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

2. Write a ratio for each situation.
   a. 310 heartbeats per 5 minutes
   b. $68 for 8 hours of work
   c. Work 40 hours in 5 days

3. A recent study shows that out of 100 pieces of a popular multicolored snack, there will usually be the following number of pieces of each color.

<table>
<thead>
<tr>
<th>Brown</th>
<th>Yellow</th>
<th>Red</th>
<th>Blue</th>
<th>Orange</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>14</td>
<td>13</td>
<td>24</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

   a. The numbers for two colors form a ratio that is equal to $\frac{7}{12}$. What are the colors? What is their ratio?
   b. If there were 500 pieces, about how many would be red?

4. Katie is making lemonade from a powder mix. The ratio of scoops of powder mix to water is 4 scoops to 1 gallon.
   a. How much water should Katie mix if she uses 12 scoops of mix?
   b. How much powder mix should Katie use if she plans to use 5 gallons of water?

5. There are a total of 60 plastic blocks. Three out of every 5 blocks are red. Is it reasonable for Briana to think there are enough red blocks to make a design that uses 32 red blocks? Explain your reasoning.

6. Which of the following expressions is not a ratio?
   A. $\frac{2}{3}$  
   B. 2:3  
   C. 2 to 3  
   D. 2 + 3

7. Which of the following compares the number of stars to the number of circles?

   A. $\frac{6}{8}$  
   B. 4:3  
   C. 3:4  
   D. 8 to 14

8. How does a ratio comparing the number of squares to the total number of shapes compare to a ratio comparing the number of arrows to the total number of shapes?

9. There are three types of animals in the pictures in Mica’s album: horses, cows, and sheep. The ratio of horses to total number of animals in the pictures is 2:8. The ratio of cows to total number of animals in the pictures is 1:4.
   a. What can you conclude about the number of horses and the number of cows in the pictures?
   b. There are 40 animals pictured in Mica’s album. How many are either horses or cows?

10. Write a ratio in lowest terms for each type of relationship for the following shapes.

   a. part-to-whole  
   b. part-to-part  
   c. whole-to-part
Lesson 17-2

11. Complete the ratio table to show ratios equivalent to 9:33.

<table>
<thead>
<tr>
<th>45</th>
<th>3</th>
<th>63</th>
</tr>
</thead>
<tbody>
<tr>
<td>330</td>
<td>132</td>
<td></td>
</tr>
</tbody>
</table>

12. Which of the following ratios is not equivalent to 9:33?

A. \(\frac{54}{198}\)  
B. \(\frac{18}{66}\)  
C. \(\frac{1}{25}\)  
D. \(\frac{6}{22}\)

13. The ratios 4:5 and \(x:80\) have a proportional relationship. What is the value of \(x\)?

A. 79  
B. 100  
C. 81  
D. 64

14. The following is a graph of the number of hours driven versus the number of miles traveled. Use the graph to answer parts a–c.

a. Is the relationship between the number of hours driven and the number of miles traveled proportional? Explain your reasoning.

b. After 3 hours of driving, how many miles would be traveled?

c. Find the value of \(x\).

\[\frac{2}{100} = \frac{x}{250}\]

MATHEMATICAL PRACTICES

Construct Viable Arguments

15. Graph the following relationship. Determine if the relationship is proportional or not proportional. Explain your reasoning.

<table>
<thead>
<tr>
<th>Number of Pens, (y)</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($), (x)</td>
<td>10</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>
Learning Targets:
- Use ratio and rate reasoning to solve problems.
- Use ratio reasoning to convert measurement units.
- Apply quantitative reasoning, including predicting and comparing, to solve real-world problems involving ratios and rates.

SUGGESTED LEARNING STRATEGIES: Close Reading, Construct Arguments, Create Representations, Identify a Subtask

Everyone tells Chris that his stories and drawings are great. Career Week is coming at his school and Chris is very excited to meet someone who has worked with movies and video game design. Chris decided to do some research before Career Week and was surprised to find out how much math is involved in filming and animation.

Animation is a series of pictures that flip by quickly in order, making something look like it is moving. A rate is a ratio that compares two quantities having different units. So, for animation, the number of pictures that go by in a second is called the frame rate, or “fps.” Two different frame rates are shown below.

60 fps

24 fps

Example A

a. Reason quantitatively. If a swimming fish is filmed at 120 frames in one second, there are 120 photos of the fish. If you played the film at 10 frames per second, how long would the film play?

Write and solve an equation using the play rate and s, the length of the film in seconds.

\[
\frac{\text{number of photos}}{1 \text{ second}} = \text{play rate} \times \text{number of seconds}
\]

\[
\frac{120}{1 \text{ sec}} = \frac{10}{1 \text{ sec}} \times s
\]

120 = 10s

120 = 10s

10

10

12 = s

Solution: The film would play for 12 seconds.
b. If Chris films the swimming fish at four times the initial speed of 120 frames in 1 second, he will have more photos of the fish. How many photos will he have with the faster filming?

Multiply the frame rate times the speed.

Number of photos = frame rate × new speed

\[
\frac{120 \text{ photos}}{1 \text{ second}} \times 4 = 480
\]

Solution: Chris will have 480 photos at the new speed.

Try These A

If a swimming fish is filmed at 100 frames in one second, there are 100 photos of the fish.

a. If you played the film at 20 frames every second, how long would the film play?

b. If Chris films the swimming fish at three times the initial speed, he will have more photos of the fish. How many photos will he have with the faster filming?

You can use *dimensional analysis* to solve problems that require one unit to be converted to another. In Example B, the *conversion factor* is found first.

Example B

Make sense of problems. How many frames would be needed for a 1-minute film if it is filmed at 1,000 frames every second?

Step 1: Determine the number of seconds in 1 minute. This is the conversion factor.

1 minute = 60 seconds

Step 2: Multiply 1,000 frames per second by 60 seconds per minute.

\[
\frac{1,000 \text{ frames}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{60,000 \text{ frames} \text{ seconds}}{1 \text{ second} \text{ minute}} = \frac{60,000 \text{ frames}}{1 \text{ minute}}
\]

Solution: There are 60,000 frames in a 1-minute movie.

Try These B

How many frames would be needed for a 1-minute film if it is filmed at 2,500 frames every second?
Example C

A bean seed is filmed as it grows. It is being filmed at a rate of 1 frame per minute. Predict how many hours of the bean’s growth will be shown in 2,910 frames.

Step 1: Determine the number of frames in 1 hour to be used as the conversion factor.

\[ 1 \text{ hour} = 60 \text{ minutes, or } 60 \text{ frames in 1 hour} \]

The conversion factor is \( \frac{1 \text{ hour}}{60 \text{ frames}} \).

Step 2: Multiply and cross out the units that are the same in the numerator and denominator.

\[
\frac{2,910 \text{ frames}}{\text{film}} \times \frac{1 \text{ hour}}{60 \text{ frames}} = \frac{2,910 \text{ frames}}{60 \text{ frames/film}} = \frac{2,910 \text{ hours}}{60 \text{ films}}
\]

Step 3: Divide 2,910 by 60 and label the solution.

\[
\frac{2,910 \text{ hours}}{60 \text{ films}} = 48.5 \text{ hours per film}
\]

Solution: 2,910 frames will produce a 48.5-hour movie of the bean’s growth.

Try These C

A rock candy crystal is filmed as it grows from a sugar solution. It is being filmed at a rate of 1 frame per hour. Predict how many days of the candy’s growth will be shown in 840 frames.
Lesson 18-1
Solve Problems Using Ratios

LESSON 18-1 PRACTICE

4. Chris found out that animators earn an average salary of $46,885 per year. He wondered how much an animator earns per month.
   a. What is the conversion factor to convert dollars per year to dollars per month? (Hint: Think about your final answer. Should it be smaller or larger than the yearly amount?)
   b. What is the average monthly salary for an animator? Show your work.

5. Chris is wondering how many hours are left until Career Week starts. It is in $2 \frac{1}{2}$ days.
   a. What is the conversion factor that will be used to convert days to hours?
   b. How many hours are in $2 \frac{1}{2}$ days? Show your work.

6. Make sense of problems. Do conversion factors always, sometimes, or never have a numerator and denominator that are equivalent? Explain your choice.

7. A flower is filmed as it goes from slowly opening in the morning to closing up at night. It is being filmed at a rate of 1 frame per 30 seconds. Predict how many frames of the flower’s blooming will be shown during 14 hours of daylight.

Check Your Understanding

1. Construct viable arguments. Explain how determining the solution for Try These C was different from determining the solution for Try These B.

2. Explain the process you use to determine how many centimeters of film are in 7 meters of film.

3. A package of film weighs $28 \frac{4}{5}$ ounces. What is the weight of the package in pounds?
Lesson 18-2
Convert Between Measurements Using Ratios

Learning Targets:
• Use ratio and rate reasoning to solve problems by reasoning about double number line diagrams and equations.
• Use ratio reasoning to convert measurement units.
• Represent mathematical and real-world problems involving ratios and rates using scale factors and proportions.

SUGGESTED LEARNING STRATEGIES: Visualization, Self Revision/Peer Revision, Discussion Groups, Sharing and Responding, Create Representations

You can use double number line diagrams to help you solve some rate problems.

Example A

Reason quantitatively. Chris may take some new photos at the beach before Career Week. Film should be stored at temperatures below 55°F. When Chris leaves for the beach, the outside temperature is 37°F. The temperature is predicted to rise 4°F every hour.

a. Predict how many hours Chris will be able to shoot photos at the beach before the temperature is too warm to store his film in the car.

Use a double number line showing temperature and hours.

Since 55°F is halfway between 53°F and 57°F, the number of hours should be halfway between 4 and 5 hours, or 4.5 hours.

Solution: Prediction: 4.5 hours

b. Confirm your prediction using dimensional analysis.

55° − 37° = 18° and \( \frac{18 \text{ degrees}}{4 \text{ degrees}} \times 1 \text{ hour} = \frac{18}{4} = 4.5 \text{ hours} \)

Solution: Chris will be able to shoot photos for \( 18° \div 4° \text{ per hour} = 4.5 \text{ hours} \).

Try These A

a. Chris often sells his photos to the local newspaper for $3 each, up to a limit of $40. Use a double number line to predict how many photos Chris can sell before the limit is reached.

b. Confirm your prediction using dimensional analysis.
Chris has to think about the sizes of photos as he converts between sizes. Sizes are measured in inches in the United States and in the metric system internationally. For instance, an 8” by 10” photo in the United States is considered to be a 203 mm by 254 mm photo in other countries.

**Example B**

The U.S. Department of State requires that a passport photo be sized so that it is a square 2 inches by 2 inches.

**a.** What is the size of the photo in millimeters?

**Step 1:** Find the conversion factor.

There are approximately 2.54 centimeters per inch and 10 millimeters per centimeter.

\[
\frac{2.54 \text{ cm}}{1 \text{ in.}} \times \frac{10 \text{ mm}}{1 \text{ cm}} = \frac{25.4 \text{ mm}}{1 \text{ in.}}
\]

There are 25.4 millimeters per inch. This is the conversion factor.

**Step 2:** Convert the dimension in inches to millimeters.

\[
\frac{25.4 \text{ mm}}{1 \text{ in.}} \times \frac{2 \text{ in.}}{1 \text{ in.}} = \frac{50.8 \text{ mm}}{1 \text{ side}}
\]

**Solution:** Each side of the photo must be about 51 millimeters.

**b.** In the passport photo, the head must be between 25 and 35 millimeters from the bottom of the chin to the top of the head. If the head is also a square, what is the minimum and maximum amount of area the person’s head must take up?

**Step 1:** Find the area of the photo in square millimeters.

Since each side of the photo is about 51 millimeters, the area is 

\[
51 \text{ mm} \times 51 \text{ mm} = 2,601 \text{ mm}^2
\]

**Step 2:** Find the area of the head in square millimeters.

Minimum area: 

\[
25 \text{ mm} \times 25 \text{ mm} = 625 \text{ mm}^2
\]

Maximum area: 

\[
35 \text{ mm} \times 35 \text{ mm} = 1,225 \text{ mm}^2
\]

**Solution:** The person’s head must take up from 625 mm² to 1,225 mm² of the photo.

**Try These B**

Chris resized a 5 in. by 7 in. photo into a passport photo. What is the original size of the photo in millimeters?
You can resize a figure by any scale factor so that the new figure is the exact same shape as the original figure.

**Example C**

Chris wants to resize an 8-inch by 10-inch photo by the scale factor \(\frac{3}{4}\).
What will be the dimensions of the new photo?

**Step 1:** Convert the 8-inch side using \(x\), the width of the new photo.

\[
\frac{3}{4} = \frac{x}{8} \quad \text{(Write a proportion using } x). \\
4x = 24 \quad \text{(Cross-multiply).} \\
\frac{4x}{4} = 24 \quad \text{(Divide each side by 4).} \\
x = 6 \quad \text{(Simplify).}
\]

**Step 2:** Convert the 10-inch side using \(y\), the length of the new photo.

\[
\frac{3}{4} = \frac{y}{10} \quad \text{(Write a proportion using } y). \\
4y = 30 \quad \text{(Cross-multiply).} \\
\frac{4y}{4} = 30 \quad \text{(Divide each side by 4).} \\
y = 7.5 \quad \text{(Simplify).}
\]

**Solution:** The new photo will be 6 inches by 7.5 inches.

**Try These C**

Chris wants to resize a 12-inch by 14-inch photo by the scale factor \(\frac{2}{3}\).
What will be the dimensions of the new photo?
Check Your Understanding

1. Chris may sell some photos of the beach online for $2 each. Use a double number line to predict how many photos he needs to sell to earn at least $21.

2. Chris wants to resize a 4-inch by 6-inch photo by a factor of \( \frac{4}{3} \). What are the dimensions of the new photo?

LESSON 18-2 PRACTICE

3. Reason quantitatively. Chris’s work was a hit during Career Week. The filmmaker told Chris that she would pay him to illustrate some of her story lines on her next project at the rate of $120 per 8-hour day.
   a. How much did Chris earn by working on the project for 20 hours?
   b. Make a graph of the relationship between the number of hours Chris worked and the amount he earned.
   c. Look at your graph. How much does Chris earn for 6 hours of work?

4. For the project, Chris included resized pictures of Career Week. He resized his 5-inch by 7-inch pictures by a factor of \( \frac{1}{2} \). What are the dimensions of the new photos?

5. Chris noticed that \( \frac{3}{4} \) of his flash drive contained photos he took during the week. His flash drive holds 16 GB. If 1 GB = 1,024 MB (megabytes), and 1 MB = 1,048,576 bytes, how many bytes were photos taken during Career Week?

6. Chris found out that \( \frac{7}{8} \) of the students in his school attended Career Day. If his school has 928 students, how many students attended that day?

7. The temperature outside school at noon on Career Day was 48°F, and it dropped 2°F each hour. If Career Day lasted 6 hours, what was the temperature outside at the end of Career Day?
Lesson 18-1

1. If a horse is filmed during a race at 100 frames in one second, there are 100 photos of the horse.
   a. If you played the film at 10 frames every second, how long would the film play?
   b. If you tripled the speed of filming, how many photos would you have with the faster filming?

2. How many frames would be needed for a 1-minute film if it is filmed at 500 frames every second?

3. A rubber ball is filmed as it bounces on a sidewalk at 25 frames every second. Predict how many seconds of film will be shown in 1,750 frames. Justify your answer.

4. How many seconds of film is in 120 minutes of video?
   A. 2
   B. 720
   C. 120
   D. 7,200

5. How many ounces are in 2.5 pounds of seeds bought to film one scene of a movie?
   A. \(\frac{5}{32}\)
   B. 30
   C. 40
   D. 400

6. The average salary for a photographer in Chris's town is $54,800 per year.
   a. What is the conversion factor to convert dollars per year to dollars per week?
   b. What is the average weekly salary for a photographer?

7. One yard of film is how many inches?
   A. 12
   B. 24
   C. 36
   D. 48

8. The filmmaker took 66 inches of film during Career Week.
   a. How many feet of film did she take?
   b. If she filmed at a rate of 400 frames per second and it took 1 minute to film 1 inch of film, how many photos did she take? Justify your answer.

9. There are two grades of students at Career Day: sixth and seventh. The sixth graders spent an average of 1.5 minutes at each booth, while the seventh graders spent an average of 2.5 minutes at each booth. Each student visited every booth and checked it off a list.
   a. How many sixth graders visited the first booth in 3 hours?
   b. What is the total number of hours spent by 400 seventh graders visiting 10 booths?

10. A typical scanning format for high-definition television is 25 frames per second, with each frame being 1,920 pixels wide and 1,080 pixels high. How many pixels are displayed in a minute?
    A. 48,000
    B. 2,880,000
    C. 2,073,600
    D. 3,110,400,000
Lesson 18-2

11. An online seller is offering photo images for $0.99 each. Use a double number line to predict how many images can be bought with $17.50.
   A. 8  B. 17  C. 18  D. 175

12. Suppose you earn $7.80 per hour. How much will you earn if you work a 20-hour week?

13. The filmmaker drove her car a distance of 250 miles to get to Chris’s school. She traveled the first 200 miles in 4 hours. At this rate, how long will it take her to make the complete trip?
   A. 1 hr  B. 4 hr  C. 5 hr  D. 5.5 hr

14. Howard made a poster for the school advertising Career Week. He first sketched his design on an 8.5 in. by 11 in. sheet of notebook paper. Then he expanded his design using a scale factor of 4.
   a. What are the dimensions of the poster?
   b. What is the area of the poster?
   c. What is the ratio of the area of the poster to the area of the sketch?

15. Suppose you resized an 8-inch by 10-inch photo to be an 11-inch by 14-inch photo. Did you use the same scale factor for each of the dimensions? Explain.

16. Your teacher taught you how to enlarge a diagram by drawing squares on the diagrams and then copying the image within each square to a larger square that has each dimension equal to 4 times the corresponding dimension of the smaller square. If the area of the original image is 32 square inches, what is the area of the enlarged image?
   A. 32 in.²  B. 64 in.²  C. 128 in.²  D. 512 in.²

17. Chris saved some of his photos on his computer tablet. The height of his tablet is 9.5 inches, and the width is 7.31 inches. What are the dimensions of the tablet in millimeters?

MATHEMATICAL PRACTICES

Attend to Precision

18. A store is advertising a new, smaller computer tablet with a height of 7.87 inches and a width of 5.3 inches. What is the area of the face of the smaller tablet in square centimeters?
Understanding Rates and Unit Rate
Zooming!
Lesson 19-1 Understanding Rates and Unit Rates

Learning Targets:
• Understand the concept of a unit rate \( \frac{a}{b} \) associated with the ratio \( a : b \) with \( b \neq 0 \).
• Use rate language in the context of a ratio relationship.
• Give examples of rates as the comparison by division of two quantities having different attributes.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Interactive Word Wall, Graphic Organizer, Self Revision/Peer Revision

For the last 28 years, students have participated in the Science Olympiad. The 2012 Science Olympiad drew 6,200 teams from 50 states. In this activity, you will use ratios and rates to describe some Science Olympiad events.

One Science Olympiad event requires teams to build a mousetrap car that both is fast and can go certain distances.

Several teams of students have decided to compete in the Mousetrap Car event. They will use mousetraps to act as the motor of their car. Seven of the students want to use wooden mousetraps, and 9 of the students want to use plastic mousetraps.

1. Write a ratio in fraction form that shows the relationship of the number of team members who want wooden traps to the number of team members who want plastic traps.

When wooden traps are compared to plastic traps, you compare different types of traps (wooden and plastic), but they have the same unit (traps). This is a ratio because the units are the same.

2. The coaches know that the students will need extra traps. These are needed so that the students can practice. Write a ratio equivalent to the one you wrote in Item 1 that shows the relationship of wooden traps to plastic traps, assuming each member will need 8 traps.
3. Use this ratio to determine:
   a. How many of each type of trap to buy.
   
   b. The total number of traps needed. Show your work.

Another way to figure out the total number of traps needed is to write a ratio comparing traps to people.

4. Write the average number of traps per 1 person as a ratio in fraction form.

   You have just written is a special type of ratio known as a **rate**. This rate shows a relationship between quantities measured with **different units** (traps and people).

   When the rate is **per 1 unit**, such as traps per 1 person, it is called a **unit rate**. Unit rates are easy to spot because they are often written with the word **per** or with a slash (/) (for example, traps per team member or traps/team member).

5. Name at least 2 other rates expressed with the word **per**.

6. Describe a situation that uses a unit rate.

   A **rate** is a comparison of two different units, such as miles per hour, or two different things measured with the same unit, such as cups of concentrate per cups of water.

   Rates are called **unit rates** when they are expressed in terms of 1 unit.

   Examples of unit rates are 60 miles per hour and 12 words per second.

7. A factory can produce small wheels for the mousetrap cars at a rate of 18,000 wheels in 3 hours. What is the unit rate per hour?

8. Use the unit rate you found in Item 4 to find the total number of traps needed for the Mousetrap Car event. Fill in the values you know.

<table>
<thead>
<tr>
<th>Unit Rate</th>
<th>Rate for Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traps/1 Person</td>
<td>Total Traps/Total People</td>
</tr>
</tbody>
</table>
   | \[
   \frac{\text{traps}}{\text{person}} = \frac{\text{traps}}{16 \text{ people}}
   \]

9. How does this compare to your answer in Item 3b? Explain.
Lesson 19-1
Understanding Rates and Unit Rates

10. Use the Venn diagram below to compare and contrast ratios, rates, and unit rates. Give an example of each in the diagram.

MATH TIP
Remember that the regions of the Venn diagram that are outside the common regions should be information that is unique to that topic.

LESSON 19-1 PRACTICE
11. Find the missing value. \( \frac{40}{8 \text{ mousetraps}} = \frac{1 \text{ mousetrap}}{} \)
12. The science teacher bought 20 mousetraps for $59.99. What was the unit cost for each mousetrap?
13. Solve: \( \frac{48 \text{ mousetraps}}{6 \text{ people}} = \frac{x \text{ mousetraps}}{1 \text{ person}} \).
14. Students spent an average of $5.50 to buy materials for the Science Olympiad. If they each built 3 mousetrap cars, what was their unit cost per car?
15. Give an example of a rate that is not a unit rate. Explain your choice.
16. Make sense of problems. Do rates always have to be expressed as a quotient? Explain how you know.
17. A recipe has a ratio of 2 cups of flour to 3 cups of sugar. How much flour is there for each cup of sugar?
18. A punch recipe has a ratio of 3 pints of sparkling water to 5 pints of fruit juice. How much sparkling water is there for each pint of fruit juice?
Another Science Olympiad event is Bottle Rockets. To compete in this event, a team must have a large supply of plastic bottles. The coaches and students decide to take advantage of specials on bottled drinks at two local stores. They will drink the contents of the bottles at their practices and meetings and use the bottles themselves to make the rockets.

Kroker’s Market:
2 bottles for $2.98
$1.59 each

Slann’s Superstore:
3 bottles for $4.35
$1.59 each

1. From the advertisements above, predict which store has the less expensive bottled drinks.

2. How can finding the unit rate for the drinks help you to determine which store to order the bottled drinks from?

3. Use the price chart for Kroker’s Market.
   a. Determine the unit price per bottle if you buy the drinks using Kroker’s 2-bottle deal.

   b. How much do the students save by using the 2-bottle deal instead of buying 2 bottled drinks at the regular price?

4. Use the price chart for Slann’s Superstore.
   a. Determine the unit price per bottle if you buy the drinks using Slann’s 3-bottle deal.

   b. How much do the students save by using the 3-bottle deal instead of buying 3 bottled drinks at the regular price?
5. **Reason quantitatively.** To decide where they will get the better deal, the students cannot simply compare unit rates. Since they need a specific number of bottled drinks, the better deal may depend on how many bottled drinks they are buying.

a. Determine how much it would cost to buy 7 bottles from Kroker’s Market. *(Hint: The students can use the deal for every 2 bottled drinks they buy, but the seventh bottle will be at regular price.)* Show your work.

b. Determine how much it would cost to buy 7 bottled drinks from Slann’s Superstore. Show your work.

c. Where should the students buy their drinks if they want to buy 7 bottles? Explain.

The students now have all of the bottles that they need. They have just a few more supplies to purchase.

One needed supply is \( \frac{1}{2} \)-inch PVC pipe to build bottle launchers for practice and competition. They do not need a specific amount of pipe, because they will use the extra pipe in the future. They want to find the best deals on this pipe by the foot.

6. The table shows rates for the cost of \( \frac{1}{2} \)-inch PVC pipe at three different wholesalers.

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Rate per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big S Supplies</td>
<td>$1.45/2 feet</td>
</tr>
<tr>
<td>Build It Again, Sam</td>
<td>$3.98/5 feet</td>
</tr>
<tr>
<td>Building Stuff</td>
<td>$1.77/2 feet</td>
</tr>
<tr>
<td></td>
<td>$28.77/50 feet</td>
</tr>
</tbody>
</table>

a. Find the unit rate for each of the prices at each of the suppliers above. Show all of your work.
b. Where should the PVC pipe be purchased? Explain why.

c. Explain why the numbers in the table make it easier to use unit rates to compare prices than using equivalent ratios.

Now, look at just the two pipe prices at Big S Supplies. When trying to decide which PVC pipe to buy at Big S Supplies, a proportion can also be used.

In this case, let $c$ represent the unknown cost of the pipe for 50 feet.

$$\frac{1.45}{2 \text{ feet}} = \frac{c}{50 \text{ feet}}$$

To determine a rule that can be used to solve for $c$, think about what you already know about solving equations.

7. Write the steps you would use in solving this proportion.

8. Construct viable arguments. How can you use this proportion to determine which is the less expensive PVC pipe at Big S Supplies? Explain your reasoning.

9. The length of a car measures 20 feet. What is the length of a model of the car if the scale factor is 1 inch:2.5 feet?
Lesson 19-2
Calculating Unit Rates

Check Your Understanding

10. The table shows rates for the cost of buying toy rocket packages. The packages cannot be broken up. What is the unit rate for each of the prices shown?

<table>
<thead>
<tr>
<th>ABC Toys</th>
<th>Z Science Supply</th>
<th>K Museum Store</th>
</tr>
</thead>
<tbody>
<tr>
<td>$49.95/5 rockets</td>
<td>$29.99/2 rockets</td>
<td>$34.95/3 rockets</td>
</tr>
</tbody>
</table>

11. Where should the teacher buy the toy rockets? Explain why.

12. Suppose the teacher wanted to buy exactly 6 toy rockets. Where should she buy them? Explain.

13. Explain why unit rates may be used to compare prices.

LESSON 19-2 PRACTICE

14. Gordon read 18 pages of a book about rockets in 40 minutes. What was the unit rate per minute? Per hour?

15. Renaldo earned $45 organizing the science section of the library. If he worked for 6 hours, what was his hourly rate of pay?

16. Make sense of problems. The price of jet fuel in North America during the last week of 2012 was recorded as $3,062 for 1,000 gallons. What was the unit price of the jet fuel?
Learning Targets:

- Use ratio and rate reasoning to solve problems.
- Represent mathematical and real-world problems involving ratios and rates using scale factors and proportions.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Predict and Confirm, Look for a Pattern

Some contestants in the Mousetrap Car event try to build the fastest car. Below are some Mousetrap Car record holders:
- One car raced 5 meters in 1.25 seconds.
- Another car raced 10 meters in 4.30 seconds.
- Another car raced 7 meters in 2.81 seconds.

1. Work with your group. Predict which of these mousetrap cars is the fastest.

2. What is the average speed of the mousetrap car that covered 5 meters in 1.25 seconds?

   To determine this, you must find the number of meters for one second. Find the meters per second by finding an equivalent ratio. Divide by 1 in the form of $\frac{1.25}{1.25}$.

   \[
   \frac{5 \text{ meters}}{1.25 \text{ seconds}} \div \frac{1.25}{1.25} = \frac{5}{1.25} \text{ meters per second.}
   \]

3. Express regularity in repeated reasoning. Another way to find the speed of each mousetrap car is to reason this way: “If 5 meters is the distance the car travels in 1.25 seconds, then how many meters does the car travel in 1 second?”
   a. Explain why a proportion can be used to find the speed of the mousetrap car. Then write and solve the proportion.

   b. Write this unit rate using the word per.

4. Confirm your prediction from Item 1 by finding the average speed as a unit rate for the other two mousetrap cars. Use one of the methods above. Round to the nearest hundredth.
5. **Reason quantitatively.** Michela used the tape diagram below to help her predict the speed of her mousetrap car. She knew that the length of the track is 15 feet, and that her car traveled one-fifth of the length of the track in 0.5 seconds.

```
track
15 feet
3 feet | 3 feet
0.5 0.5 0.5 0.5 0.5
```

a. What was the unit speed of Michela's mousetrap car?

b. How can you use the tape diagram to write Michela's mousetrap car as a unit rate?

6. Another car in the Mousetrap Car event raced 12 meters in 5.2 seconds. Is it faster than Michela's car? Explain. Note: 1 ft = 0.305 m.

7. What is the average speed of a mousetrap car that covers 4 meters in 2.5 seconds?

**Lesson 19-3 Practice**

8. What is the average speed of a mousetrap car that covers 5.25 meters in 1.75 seconds?

9. How can finding the unit rate help you determine the fastest car in the Mousetrap Car event?

10. **Reason quantitatively.** Michela experimented with filming her toy car as it ran through the racecourse. Her film was made up of 150 photo frames taken in 5 seconds.

   a. What is the unit rate for the frame speed?
   
   b. If all contestants in the race filmed their cars, how can you use the frame speed to determine which car was the fastest?
11. Solve the proportion. Show your work.
\[
\frac{7.375 \text{ meters}}{1.475 \text{ seconds}} = \frac{x \text{ meters}}{1 \text{ second}}
\]

12. How can you use a proportion to show which mousetrap car is the fastest?

13. Caroline was in charge of buying the mousetraps for the event. She had to choose whether to buy 12 mousetraps in a package that costs $39.99 or 15 mousetraps in a package that costs $45.99.
   a. What is the unit rate for each of the packages of mousetraps?
   b. Which package was a better buy? Explain.
   c. Describe a situation for which Caroline should buy mousetraps in packages of 15.

14. Suppose Caroline needs to buy only 10 mousetraps. Which package should she purchase? Explain.

15. **Use appropriate tools strategically.** Describe how Caroline could use a tape diagram to help her decide which package to buy in Item 13.
ACTIVITY 19 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 19-1

1. A factory can produce mousetraps at a rate of 1,500 traps in 3 hours. What is the unit rate per hour?

2. A factory that produces toy wheels finds that they must discard 3 out of every 100 wheels because they are defective. How many wheels would they expect to discard every day if the factory produces 1,200 wheels per hour in each 8-hour day?

3. A mousetrap car is filmed as it runs the racecourse. The film includes 1,200 photo frames taken in 8 seconds. How many frames were taken in one second? Justify your answer.

4. Last year, one science teacher spent $65.50 to buy materials for the Mousetrap Car event. If 12 students participated in the event, what was the teacher’s unit cost per student?
   - A. $2.98
   - B. $6.55
   - C. $5.46
   - D. $7.86

5. Mr. Walker, the contest sponsor, bought toy wheels in a package of 6 dozen for $81.00. What was the unit cost of each wheel?
   - A. $1.125
   - B. $1.50
   - C. $6.75
   - D. $13.50

6. Mr. Walker wrote notes about the event that would help him plan for the following year. If he will still purchase 8 traps per person, how many traps will he need to purchase for 25 people in the Mousetrap Car race?
   - A. 4
   - B. 8
   - C. 200
   - D. 400

7. A meatball recipe has a ratio of 2 cups of breadcrumb mixture to 3 pounds of ground meat. How much breadcrumb mixture is there for 1 pound of ground meat?

Lesson 19-2

8. The table shows rates for the cost of buying model rocket packages. What is the unit rate for each of the models shown?

<table>
<thead>
<tr>
<th>XYZ Toys</th>
<th>AAA Science Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$69.95/5 models</td>
<td>$59.95/4 models</td>
</tr>
</tbody>
</table>

9. PVC pipe used to make the toy rockets is sold by the foot. If 8 feet cost $16.89, what is the cost for 30 feet of pipe sold at the same unit rate?
   - A. $2.11
   - B. $4.50
   - C. $63.34
   - D. $506.70

10. Write the steps you would use in solving the proportion.
    \[
    \frac{12.50}{4 \text{ feet}} = \frac{c}{24 \text{ feet}}
    \]

11. The height of the Atlas V rocket used for space lifts was 205 feet. What is the height of a model of the Atlas V rocket if the scale factor is 1 ft : 32 ft?
    - A. 6.4 ft
    - B. 12.8 ft
    - C. 102.5 ft
    - D. 6,560 ft
Lesson 19-3

12. An online seller is offering 30 photo images of mousetraps for $24. Use a tape diagram to predict the unit cost of each image.
   a. $0.72  
   b. $0.80  
   c. $1.20  
   d. $1.25

13. Mr. Walker, a runner, asked students to find the unit rate of the winner of the first Boston Marathon in 1897. John J. McDermott ran the marathon in 175 minutes. The length of the course was only 24.5 miles instead of 26 miles as it is today. What was the unit rate?

14. Mr. Walker used proportions and the following example to teach students how to calculate speeds: An F-15 Eagle travels at a speed of 1,875 miles per hour for 3.5 hours. Which distance solves the proportion he used?
   A. 535.7 mi  
   B. 3,750 mi  
   C. 4,687.5 mi  
   D. 6,562.5 mi

15. Jackie made a poster for the school advertising the Science Olympiad. She included sample statistics on the speed of the mousetrap cars from the previous years.
   a. If one car raced 8 meters in 1.5 seconds, what was its speed?
   b. How does this compare to the record time of 10 meters in 2.32 seconds?

16. Mr. Walker drove 20 miles to pick up supplies for the Science Olympiad. If the trip to the supply store took 0.75 hours, what was his unit speed for one hour?

17. Bryce, a previous winner of the contest, made a trip of 360 miles in 6.5 hours. At this same average rate of speed, how long will it take Bryce to travel an additional 300 miles so that he can judge the contest? Explain your reasoning.

18. It is about 2,508 miles from a Science Olympiad in Orange County, California, to a Science Olympiad in Orange County, Florida. With an average speed of 70 miles per hour, about how long will it take to drive from one to the other? Use a proportion in determining your answer.

19. One Mousetrap Car contestant researched the speed of actual race cars. He found that when NASCAR drivers race on the Phoenix International Raceway, they make 312 laps. In April 2009, the race that was held there lasted for about 3 hours. What was the approximate rate the racers were traveling?

MATHEMATICAL PRACTICES
Reason Quantitatively

20. On the way to the Mousetrap Car contest, the judge drove from Exit 32 on the highway to Exit 170 in 2 hours. Exits 32 and 170 are 138 miles apart. Did the judge follow the speed limit of 65 mph? Explain how you know using a proportion.
Wendy has a summer job working 5 days per week. She is surprised how many decisions she has to make. Her decisions are shown in the questions below.

1. The two different pay options she may choose from are either $62 per day or $304 per week. Which is the better deal for Wendy? Use unit rates to explain your decision.

2. Using the option you chose in Item 1, determine how much money Wendy will earn by working 4 weeks.

3. To get the right color to paint the house, Wendy must mix 1 gallon of green paint with 3 gallons of white paint.
   a. Write a ratio in 3 different ways to show the relationship between green paint and white paint.
   b. How many gallons of paint will her mixture make?

4. Wendy is told ahead of time that she will need to purchase about 12 gallons of paint in order to cover the entire house.
   a. Write equivalent ratios to determine the amount of green and white paint she will need to purchase.
   b. If there are 4 quarts in 1 gallon, how many quarts of paint does she need to purchase?

5. How many gallons of green paint would be needed if Wendy had 10 gallons of white paint? Explain your reasoning.

6. How many gallons of white paint would Wendy need to mix with 0.5 gallon of green paint? Explain your answer.

7. Suppose that Wendy accidentally mixed 2 gallons of green paint with 3 gallons of white paint.
   a. How would her color change? Would it be darker or lighter? Explain.
   b. Without starting over, how could she fix her mistake to get the right color to paint the house?
## Scoring Guide

The solution demonstrates these characteristics:

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4a-b, 5, 6, 7a-b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Clear and accurate understanding of ratios, unit rates, and solving proportions.</td>
<td>• An understanding of ratios, unit rates, and solving proportions that usually results in correct answers.</td>
<td>• An understanding of ratios, unit rates, and solving proportions that sometimes results in correct answers.</td>
<td>• Incorrect or incomplete understanding of ratios, unit rates, and solving proportions.</td>
<td></td>
</tr>
<tr>
<td>• Effective understanding and accuracy in converting between measurements.</td>
<td>• Mostly correct conversion between measurements.</td>
<td>• Difficulty converting between measurements.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem Solving (Items 1, 2, 3, 4b, 5, 6, 7b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• An appropriate and efficient strategy that results in a correct answer.</td>
<td>• A strategy that may include unnecessary steps but results in a correct answer.</td>
<td>• A strategy that results in some incorrect answers.</td>
<td>• No clear strategy when solving problems.</td>
<td></td>
</tr>
<tr>
<td>• Accurate interpretation of the solution of a proportion to solve a problem.</td>
<td>• Interpretation of the solution of a proportion to solve a problem.</td>
<td>• Difficulty interpreting the solution of a proportion to solve a problem.</td>
<td>• Incorrect or incomplete interpretation of the solution of a proportion to solve a problem.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Modeling / Representations (Items 1, 3a, 4a, 5, 6, 7b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Accurately representing a problem situation with a ratio, proportion, or unit rate.</td>
<td>• A mostly correct representation of a problem situation with a ratio, proportion, or unit rate.</td>
<td>• Difficulty representing a problem situation with a ratio, proportion, or unit rate.</td>
<td>• An incorrect or incomplete representation of a problem situation with a ratio, proportion, or unit rate.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning and Communication (Items 1, 5, 6, 7a-b)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Precise use of appropriate math terms and language to explain solutions using ratios and proportions.</td>
<td>• An adequate explanation of solutions using ratios and proportions.</td>
<td>• A misleading or confusing explanation of solutions using ratios and proportions.</td>
<td>• An incomplete or inaccurate description of solutions using ratios and proportions.</td>
<td></td>
</tr>
</tbody>
</table>
Using Models to Understand Percents

A “Cent” for Your Thoughts

Lesson 20-1 Using Models to Understand Percents

Learning Targets:
• Find a percent of a quantity as a rate per 100.
• Represent ratios and percents with concrete models and decimals.
• Represent benchmark fractions and percents.
• Generate equivalent forms of decimals and percents.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Marking the Text, Visualization, Quickwrite, Create Representations, Simplify the Problem

Another way to represent a part-to-whole relationship is by using another type of ratio called a percent. A percent is a ratio that is always a number compared to 100. The symbol % is used to represent the term percent.

1. Consider the words century, cent, centavo, and centimeter. What do these words have in common?

2. What other words do you know that have the base word cent in them?

3. Consider the parts of the word percent. Why do you think a number out of 100 is called a percent?

4. Reason quantitatively. Since you know that there are 100 cents in a dollar and percents are parts of 100, write each of these dollar amounts as a percent.
   a. a penny
   b. 10 cents
   c. $0.25
   d. 5 cents
   e. a dollar
   f. $1.50

Percent means parts per hundred. A percent can be expressed as a fraction, such as 87/100, or with a percent sign, 87%.
5. Use the grid to answer the following questions.

a. How many squares out of 100 are shaded? _____ out of _____

b. Replace out of 100 with the word percent:

c. Replace percent with its symbol:

6. Since percents are parts of 100, they can be modeled on a 10-by-10 grid.
   a. Create a design using red, orange, yellow, green, and blue. Be sure to color in all of the squares.

   b. In the table below, write the percent of the grid that is covered by each color.

<table>
<thead>
<tr>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of Grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. Represent the percent for each color using a strip diagram.

   d. Add together the percents from the table above. What do you notice about the sum?

   e. How is your answer to part c related to what you know about percents?
Lesson 20-1
Using Models to Understand Percents

7. There are some important benchmark percents that will be seen often in math class and in everyday life. Use the grids to determine the percent that represents each fraction.

a. \( \frac{1}{2} \)

b. \( \frac{1}{4} \)

c. \( \frac{1}{10} \)

d. \( \frac{1}{5} \)

Equivalent forms of decimals and percents can sometimes be used to represent real-world problems.

8. Make sense of problems. Out of 100 students in the cafeteria, 42 wanted chicken fingers and 24 wanted salad. Explain how you can represent the number of students who did not want either choice as a decimal and as a percent.

9. A common tip for a restaurant bill is 15%. Explain how much money that adds to the amount you pay.

10. Write a description of a math context that involves money that can be expressed using decimals or percents. Be sure to use appropriate vocabulary, both real-world and mathematical, to describe the situation. Refer to the Word Wall as needed to help you choose words for your description.
Lesson 20-1
Using Models to Understand Percents

Check Your Understanding

11. Write the shaded part of each figure as a percent.
   a. 
   b. 

12. Write each amount as a percent.
   a. \( \frac{12}{100} \)
   b. 79 out of 100

13. Abby received an 80% on her spelling test. Tell what this means.

LESSON 20-1 PRACTICE

14. Copy and complete the table below by filling in missing percents or shading figures to represent given percents.

<table>
<thead>
<tr>
<th>Model</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Model" /></td>
<td>100%</td>
</tr>
<tr>
<td><img src="image2" alt="Model" /></td>
<td>10%</td>
</tr>
<tr>
<td><img src="image3" alt="Model" /></td>
<td>100%</td>
</tr>
<tr>
<td><img src="image4" alt="Model" /></td>
<td>10%</td>
</tr>
</tbody>
</table>

15. Describe how you should write a percent for the shaded part of a figure that has 20 equal squares with 8 squares shaded and 12 squares unshaded.

Use a grid to help you write each benchmark fraction as a percent.

16. \( \frac{1}{5} \)
17. \( \frac{1}{4} \)
18. \( \frac{3}{4} \)
19. \( \frac{1}{8} \)

20. Model with mathematics. A typical professional basketball player may make 64 out of 100 free throws. Draw a model to show this ratio. Then write the ratio as a percent.
Lesson 20-2
Percents, Fractions, and Decimals

Learning Targets:
• Represent ratios and percents with fractions and decimals.
• Represent benchmark percents such as 1%, 10%, 25%, 33 1/3%, and multiples of these values using number lines and numbers.
• Use per cents, fractions, and decimals to show parts of the same whole.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Note Taking, Sharing and Responding, Create Representations

1. Color the grid. The table at the right tells how many squares to fill with each color. Make any design you want.

<table>
<thead>
<tr>
<th>Color</th>
<th>Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>40</td>
</tr>
<tr>
<td>Orange</td>
<td>8</td>
</tr>
<tr>
<td>Yellow</td>
<td>13</td>
</tr>
<tr>
<td>Green</td>
<td>17</td>
</tr>
<tr>
<td>Blue</td>
<td>22</td>
</tr>
</tbody>
</table>

2. For each color, write a ratio of the number of squares of that color to the total number of squares using a colon. Then write each ratio in fraction, decimal, and word form and as a percent.

<table>
<thead>
<tr>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio (:)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decimal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Word Form</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Reason quantitatively. Use the table from Item 2 to answer each question.
   a. What is the sum of the percents?

   b. What is the sum of the fractions?

   c. What is the sum of the decimals?

   d. What relationships do you see among your answers to parts a–c?

Recall that one way to convert a fraction to a decimal is by division.
For example, $\frac{3}{4}$ is 3 divided by 4, which gives a quotient of 0.75. This can be written as a percent, 75%.
This gives the same answer as using equivalent fractions:
$\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$. 

MATH TIP

Activity 20 • Using Models to Understand Percents
Lesson 20-2
Percents, Fractions, and Decimals

4. Look at the table showing the colors you used in the grid.
   a. List the colors and percents from Items 1 and 2 in order from the color most used to the color least used.

   b. What representations other than the percents could you have used to order the colors?

5. What about the grid in Item 1 made it easy to find the percent?

6. How many tiles make up the message Hi! as shown?

7. To find the percent of the tiles in Hi! that are in the H, first find either the fraction or the decimal that represents the number of tiles in the H out of the total number of tiles.
   a. Which is easier to find in this situation, a decimal or a fraction? Explain.

   b. Find the equivalent fraction to your answer in hundredths, since percent is a number out of 100. Then convert the hundredths to a percent.

   c. Write this percent as a decimal.

8. Think about the tiles in the letter i.
   a. What percent of the tiles in Hi! are in the i?

   b. Write this percent as a decimal.

9. Use your answers to Items 7 and 8 to determine what percent of the tiles in Hi! are in the ! without counting them. Explain how you found your answer.
10. Write the percent from Item 9 as a decimal and as a fraction.

11. You just learned to write percents using a ratio or a decimal written in hundredths. Convert each fraction, decimal, or ratio below to a percent. If not already in hundredths, first convert to hundredths and then write as a percent.
   a. \(0.45\)  
   b. \(\frac{34}{100}\)  
   c. \(0.9\)  
   d. \(\frac{7}{10}\)  
   e. \(\frac{11}{25}\)  
   f. \(0.30\)

In the last activity you learned that there are some fraction, decimal, and percent conversions that are commonly used and are called benchmarks. Solving problems will be easier if you learn and remember them.

12. a. **Express regularity in repeated reasoning.** Complete the table below with the fraction, decimal, and percent forms of these commonly used numbers.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{4})</td>
<td></td>
<td>25%</td>
</tr>
<tr>
<td>(\frac{1}{3})</td>
<td></td>
<td>33%</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

b. Place the fractions, decimals, and percents on this triple number line.

- Fractions: \(\frac{1}{4}, \frac{1}{3}, 1\)
- Decimals: 0.1, 0.333...
- Percents: 25%, 33\%, 50%

1% 20% 50% 75%

1

12. c. What patterns do you notice in the table and the number line that can help you to remember the different forms of these numbers?
13. Work with your group. Use the grid below. When answering parts a–c below, do not use more than one color in a box. Assign each group member a region to color from parts a–c.

- Color 36% of the grid blue. Write the fraction and the decimal that represent the amount of the grid that is now blue.

- Color \( \frac{2}{5} \) of the grid red. Write a decimal and the percent to represent the number of red boxes.

- Color 0.16 of the grid yellow. Write this amount as a fraction and convert your fraction to a percent.

- What percent of the grid is now shaded? Write this percent as a decimal and a fraction.

14. Use the squares you colored in on the grid to order 36%, \( \frac{2}{5} \), and 0.16 from least to greatest.

15. If you did not have a shaded model to look at, you could use a number line to compare percents, fractions, and decimals. Place 36%, \( \frac{2}{5} \), and 0.16 on the number line below.

16. Use this figure: 

   - What percent of the figure is shaded? Explain how you determined your answer.

   - How is this percent different from the other percents you have found in this activity?

   - How would you read this percent? Write your answer in words below.
Lesson 20-2
Percents, Fractions, and Decimals

Check Your Understanding

17. Write 55% as a decimal and as a fraction.
18. Kate kicked 25 goal shots at soccer practice and scored on 13 of them. What percent of shots did she make?
19. Explain why fractions may represent a quantity better than a percent.

LESSON 20-2 PRACTICE

Replace each bold number in the facts below with a percent.

20. \( \frac{1}{4} \) of all the bones in your body are in your feet.
21. About 0.18 of people let their pets sleep in their beds.
22. About \( \frac{8}{20} \) of America is wilderness.
23. Pizzerias make up about \( \frac{1}{2} \) of all restaurants.
24. Reason abstractly. Copy and complete the table below by filling in missing amounts and shading figures. Write ratios using a colon (:) to represent part-to-whole relationships.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Ratio</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3:5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{1}{4} )</td>
<td>0.3</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>7:10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

25. Write each number as a percent.
   a. \( \frac{2}{3} \)  
   b. 0.23  
   c. \( \frac{73}{100} \)
Learning Targets:
- Find a percent of a quantity as a rate per 100.
- Generate equivalent forms of fractions, decimals, and percents using real-world problems.
- Represent percents with concrete models, fractions, and decimals.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Note Taking, Think-Pair-Share, Critique Reasoning, Sharing and Responding, Create a Plan, Construct an Argument

To convert percents that include tenths to fractions, the decimal point must be moved within the fraction so that there is no decimal point in either the numerator or the denominator.

1. a. Explain how you write a percent as a fraction. What is 51.2% written as a fraction?

b. Fractions should not have decimal points in the numerator. How can the decimal point be eliminated while still keeping this an equivalent fraction?

c. What fraction is equivalent to 51.2%?

2. Percents are commonly used in trivia or fun facts. Convert each percentage in the facts below to decimals and fractions.
   a. About 50.8% of the U.S. population is female.

   b. In the U.S., 32.4% of households own a cat.

3. Find four examples of percents used in real life. You may use newspapers, signs, pictures, or another source. Create a poster showing the percents, giving their equivalent decimal and ratio forms, and telling what the percents mean in the situation. Share your poster with your class, and describe how you organized its contents.
Lesson 20-3
More Percents, Decimals, and Fractions

4. **Reason quantitatively.** A factory produces stickers at a rate of 4,000 sheets per minute. They know that 1.52% of the sheets of stickers are rejected because at least one sticker is loose on the sheet.
   a. Express the percent rejected as a rate per 100.
   b. Write and solve a proportion to find how many sheets are rejected each minute during production.
   c. How many whole sheets are rejected? Write your answer as a ratio.

5. **Make sense of problems.** In 2012, 40.1% of the population of China, about 1,343,000,000 people, were Internet users.
   a. Express the percent as a rate per 100 people in China.
   b. About how many people in China use the Internet?
   c. If the number in part b represents 22.4% of the Internet users in the world, predict the number of Internet users there were in the world in 2012. Justify your reasoning.

**Check Your Understanding**

6. Use what you have learned about converting percents, decimals, and fractions to each of the different forms. Then compare each amount.
   a. \( \frac{5}{7} \) \( 71\% \)
   b. 0.5625 \( 56.4\% \)
   c. 27\% \( 0.3 \)
   d. 10\% \( 0.01 \)

7. Write 89.6\% as a decimal and as a fraction.

8. Put the following amounts in order from greatest to least: 60\%, \( \frac{2}{3} \), 0.599. Show the form you choose to convert the numbers in order to compare them.
LESSON 20-3 PRACTICE

9. Order from greatest to least: 43%, $\frac{3}{7}$, 0.453.

10. What fraction is equivalent to 123.5%?

11. Carlos has $10 more than Jeremy. Jeremy has $5 more than Michele. Altogether they have $80. What part of 100 does Michele have?

12. Explain how you would write $\frac{7}{8}$ as a rate per 100.

13. **Reason quantitatively.** A factory produces bottled water at a rate of 2,000 cases per hour. They know that 1.14% of the cases must be rejected because at least one bottle was damaged in the production line.
   a. Express the percent as a rate per 100.
   b. Write and solve a proportion to find how many cases are rejected each hour during production.

14. **Model with mathematics.** In 2012, a survey found that 92% of people in the 18–29 age group used social networking sites.
   a. Express the percent as a rate per 100 people.
   b. What is this percent written as a fraction?

15. **Model with mathematics.** In 2012, there were about 620,000,000 websites in the world. About $\frac{2}{3}$ of these websites were inactive for various reasons. What percent of the websites were inactive?
ACTIVITY 20 PRACTICE

Write your answers on notebook paper. 
Show your work.

**Lesson 20-1**

1. Write the shaded part of each figure as a percent.  
   a. 
   b. 

2. Write each amount as a percent.  
   a. \( \frac{17}{100} \)  
   b. 23 out of 100

3. Marco’s mother told him that she would add 25% to his allowance if he saved it all. What fraction is this?

4. In 2012, 75% of the population of India had mobile phones in use. Draw a model to show this percent. Then write the percent as a ratio.

5. A dairy that produces milk cartons for schools finds that they must discard 4 out of every 100 cartons because they are not sealed properly. What percent of the cartons of milk are not discarded?  
   A. 2%  
   B. 40%  
   C. 4%  
   D. 96%

6. Three-fourths of the students in sixth grade participate in after-school activities. What percent is this?  
   A. 3%  
   B. 25%  
   C. 50%  
   D. 75%

**Lesson 20-2**

7. Convert each fraction, decimal, or ratio below to a percent. If not already in hundredths, first convert to hundredths and then write as a percent.  
   a. \( \frac{7}{20} \)  
   b. 91:100  
   c. 0.34

8. Which letter on the triple number line below corresponds to \( \frac{1}{2} \) ?  
   
   Fractions: A  B  C  D
   Decimals: 0.1 0.5 1
   Percents: 20% 75%
   A. A  
   B. B  
   C. C  
   D. D

9. Order 68%, \( \frac{3}{5} \), and 0.72 from least to greatest.

10. What fraction of each figure below is shaded? Explain how your determined your answer. Then give the percent for each fraction.  
   a.  
   b. 

11. In 2011, over 82% of the population of Mexico used a mobile phone. What fraction is equivalent to 82%?  
   A. \( \frac{18}{100} \)  
   B. \( \frac{19}{50} \)  
   C. \( \frac{41}{50} \)  
   D. \( \frac{83}{100} \)
Lesson 20-3

12. What is 68.2% written as a fraction in lowest terms?
   A. \( \frac{8}{25} \)      B. \( \frac{159}{500} \)
   C. \( \frac{17}{25} \)      D. \( \frac{341}{500} \)

13. About 78.6% of the population of North America used the Internet in 2012. Convert the percent to a decimal and a fraction in lowest terms.

14. Put the following amounts in order from greatest to least.
   54%, \( \frac{4}{7} \), 0.525
   A. \( \frac{4}{7} \), 54%, 0.525      B. \( \frac{4}{7} \), 0.525, 54%
   C. 0.525, 54%, \( \frac{4}{7} \)      D. 0.525, \( \frac{4}{7} \), 54%

15. A factory produces toy dolls at the rate of 400 per hour and has to recycle 3% of them because the clothes are torn.
   a. Express the recycle percent as a rate per 100.
   b. How many dolls are recycled each hour during production?

16. The number of websites in 2012 represented an increase of 28% over 2011. Explain how you write 28% as a rate per 100.

17. In 2012, the number of mobile phones in the United States was 103.9% of the population.
   a. Explain how you write the percent as a fraction.
   b. Give a reason for why this percent can be over 100.

18. Gina traveled 48% of the distance from her home in Maryland to Chicago in one day. Represent the percent using the model below.

19. The surface of the Earth is about 70% water.
   a. What does the percent 30% represent?
   b. Write 30% as a fraction.

MATHEMATICAL PRACTICES
Model with Mathematics

20. Shade in 70% of the counters below. What fraction of the counters are shaded?
Learning Targets:

• Solve real-world problems to find the percent given the part and the whole.
• Use ratio and rate reasoning to solve real-world and mathematical problems.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Note Taking, Quickwrite, Identify a Subtask

Isaac and his older brother, Nate, both need to find part-time jobs. Nate is considering starting a deejay business and wants Isaac to be his partner. They could share the deejay jobs, advertising, scheduling, and billing.

The brothers did some research and found prices for startup equipment. The items they needed for their business and the cost for each item are shown in the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fog machine</td>
<td>$150.00</td>
</tr>
<tr>
<td>Controller for MP3 and computer</td>
<td>$280.00</td>
</tr>
<tr>
<td>Speakers</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>Mirrored ball and light</td>
<td>$70.00</td>
</tr>
<tr>
<td>Wireless microphone</td>
<td>$250.00</td>
</tr>
<tr>
<td>Lights</td>
<td>$750.00</td>
</tr>
</tbody>
</table>

1. How much money will be needed to start the deejay business? Show your work.

The boys asked their parents for a loan to start the business. Their parents told them that they would consider loaning them the money if they put together a business plan showing their costs and their expected income.

The first step for the boys is to determine what percent each item above is of their total budget. Nate had already learned how to determine percents in school, but Isaac had not learned yet. Nate explained to his brother the process they could use to find these percents.

A proportion can be used to find a percent of a number. When using a proportion to find a percent, information from the problem is used to set up the two ratios that form the proportion.
Lesson 21-1
Using Models to Understand Percents

This is the example that he showed Isaac.

**Example A**
25 is what percent of 80?

**Step 1:** Set up the proportion.

\[
\frac{x}{100} = \frac{25}{80}
\]

The percent is always over a denominator of 100 since percents are out of 100.

25 is the numerator because it is part of the amount you are working with.

80 is the denominator because it is the whole amount you are working with.

**Step 2:** Solve the proportion.

\[
80x = 25 \cdot 100 \\
80x = 2,500 \\
\frac{80x}{80} = \frac{2,500}{80} \\
x = 31.25
\]

**Solution:** When rounded to the nearest whole percent, 25 is 31% of 80.

**Try These A**
Find the percent the fog machine is of the total budget. Write the proportion used to solve the problem and show any work needed to solve the proportion. Round to the nearest whole percent.

2. Find the percents the other items are of the total budget. Round to the nearest whole percent. What is the sum of the percents?
Lesson 21-1
Using Models to Understand Percents

3. When finding percents, can the answer ever be greater than 100%? Give an example illustrating why or why not.

Check Your Understanding

4. 18 is what percent of 96?
5. Isaac wanted to earn $200 of the startup equipment cost for the deejay business. What percent of $2,500 is this?
6. The boys’ aunt wanted to contribute $2,100 toward the cost. What percent of $2,500 is this?

LESSON 21-1 PRACTICE

7. $300 is what percent of $3,200?
8. Construct arguments. Suppose Nate and Isaac wanted to add another item to their list of startup equipment. Use an example to describe how you would find the percent this item’s cost is of the total budget.
9. Nate and Isaac decided they also needed a work schedule to accommodate their deejay business. They planned to spend 12 out of 20 work hours per week performing as deejays. What percent is this?
10. Isaac predicted that advertising their business would add an additional $400 out of the $900 the brothers were adding to the equipment cost. What percent is $400 out of the additional amount they were adding?
11. Isaac’s parents paid $3,200 per month to rent an office space for their own business. Of this amount, $352 was for utilities. What percent of their rental cost was for utilities?
12. Use a proportion to determine what percent $1,800 of the $2,500 startup cost is.
13. Reason quantitatively. Nate found sound amplifiers online for $150 off the price of $990, although he did not buy them. About what percent would he have saved on the speakers if he had bought them?
Learning Targets:

- Solve real-world problems to find the part, given the whole and the percent.
- Use ratio and rate reasoning to solve real-world and mathematical problems.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarizing, Create a Plan, Identify a Subtask

The boys’ parents decided the business would be a good investment and loaned them the money. However, Nate and Isaac would have to pay interest when they paid the loan back in one year.

1. When dealing with interest, how can you determine if the interest is going to be paid to you or if it will be paid to someone else?

Simple interest on a loan for one year can be determined using a proportion. If you know the percent and the total, you can find the part. Remember to set up the proportion using the information you know from the problem.

Example A

Find the interest for a 2% interest rate loan of $3,000 for 1 year.

Step 1: Set up a proportion.

\[
\frac{2}{100} = \frac{x}{3,000}
\]

This time we know the percent. It is 2%. The percent is always the numerator over 100 since percents are out of 100.

$3,000 is in the denominator because it is the whole amount of the loan.

Step 2: Solve the proportion.

\[
100x = 2 \times 3,000 \\
100x = 6,000 \\
x = \frac{6,000}{100} \\
x = 60
\]

Solution: The interest is $60 on a $3,000 loan at a rate of 2% for 1 year.
Lesson 21-2
Find the Part Given a Percent and the Whole

Try These A

a. How much will the interest be for a $500 loan at 6% interest for one year?

b. If 40% of 240 minutes of music are slow songs, how many minutes of slow songs will there be?

c. The future deejays know that they cannot expect that all customers will give them a top rating. If 85% of the customers are extremely happy with their work, how many customers out of 120 should they expect to be extremely happy?

d. The loan that Nate and Isaac got from their parents was at 2% interest for one year. How much will the boys pay in interest on their loan of $2,500? Show your work.

Nate and Isaac have now picked out and purchased equipment. Next, they need to set their prices. To do this, they decide to look at advertisements for other deejay businesses and find an average price. They decide to charge $649 for 4 hours of service. In addition, they are going to charge $200 for each hour that they work beyond 4 hours.

2. How much would an event that lasted 6 hours cost the customer? Show your work.

After the first month, the brothers found that they were not getting as many jobs as they thought they would. They decided to offer a summer discount to get more business.

3. a. Construct viable arguments. Is a discount added to or subtracted from the total? Explain your thinking and give a real-life example.
b. What is the difference between a $25 discount and a 25% discount?

4. Use a proportion to determine what 15% of the $649 base rate is.

5. Explain how to find the discounted amount they will offer their customers. What is the discounted amount?

Nate and Isaac have gotten quite a few jobs using their discounted price. In fact, with the holiday season approaching, they feel they can raise their price above the original price of $649 for 4 hours.

6. They decide to mark up the price of $649 by 10%.
   a. What is a markup?

   b. Give an example of a markup that you have seen on clothing or other items.

   c. Use a proportion to find the amount that the price will be marked up.

   d. Using the original amount and the markup you just found, determine their new price for 4 hours of work.

7. How are markups and discounts the same and different? You may use a graphic organizer to help show your thinking.
Lesson 21-2
Find the Part Given a Percent and the Whole

Check Your Understanding

8. Find 45% of $649.

9. Isaac found a wireless microphone online at a sale price of 32% off.
   a. What is 32% of $250.00?
   b. What is the final price after the discount?
   c. Describe another way you could calculate the final price using a different percent.

10. Nate’s mother wanted to buy some jewelry until she found out it was marked up 400%. If the original jewelry cost $100, what was the final price of the jewelry?

11. Explain why markup is necessary for retail sales.

LESSON 21-2 PRACTICE

12. What is 108% of 112?

13. Draw a group of 15 identical music CDs or other simple figures. Shade 60% of the figures. Explain how you know that you shaded 60%.

14. Isaac played a video game 20 times and won about 70% of the games. How many games did he win?

15. Nate tells his mom that he took a test with 60 questions and scored 85%. How many questions did he answer correctly? Show how you know.

16. Reason abstractly. In a survey of 398 students, 52% said they loved music. Use estimation to explain about how many students loved music.

17. Make sense of problems. Which costs less to buy, a $1,000 computer that is discounted 20% and then offered at an additional 10% off, or a $1,000 computer that is discounted 30%? Explain.
Learning Targets:

- Solve problems to find the whole given a part and the percent.
- Represent ratios and percents with fractions and decimals.
- Represent benchmark percents such as 1%, 10%, 25%, and \( 33\frac{1}{3}\% \), and multiples of these values using number lines and numbers.
- Use equivalent percents, fractions, and decimals to show parts of the same whole.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Graphic Organizer, Note Taking, Identify a Subtask

Nate and Isaac need to have access to thousands of songs. They both have MP3 players and are using them to create playlists for the business. Nate’s MP3 player shows that he has used 53% of the memory. He can’t quite remember how much his MP3 player holds, but knows that he can find out using a proportion.

Example A

There are 9.4 hours of music left on Nate’s playlist, and this is 47% of the total memory. How much memory does Nate’s MP3 player have?

Step 1: Set up a proportion.

\[
\frac{47}{100} = \frac{9.4}{x}
\]

The part is 9.4 because that is the part of the memory remaining.

The variable is in the denominator because we do not know the whole (total) amount of memory of the MP3 player.

Step 2: Solve the proportion.

\[
47x = 9.4 \cdot 100
\]

\[
47x = 940
\]

\[
\frac{47x}{47} = \frac{940}{47}
\]

\[
x = 20
\]

Solution: Nate’s MP3 player can hold 20 hours worth of playlists.
Lesson 21-3
Find the Whole Given a Part and the Percent

Try These A
Make sense of problems. Using a decimal equivalent to the percent, write another equation you could have solved to find the total memory of Nate’s MP3 player.

1. Set up and solve a proportion.
   a. Nate worked for 28 hours last week. This is 40% of the total hours he worked this month. How much did he work this month?

   b. Isaac’s part of the check at dinner was $18.00 and he was paying 25% of the total bill. How much was the total bill?

   c. The brothers’ music project at school was worth 44 points. This is 88% of the total number of points possible. Using a decimal equivalent to the percent, write an equation to find the total number of points possible on the project. Then solve the equation.

Check Your Understanding

2. Are tips added or subtracted from the total bill? Explain.

3. Nate paid $108 for a food bill that included a 15% tip. Using a decimal equivalent to the percent, write an equation to find the total amount of the bill. Then solve the equation.

4. Some of the students at Beats Middle School say that proportions make working with percents easier. Do you agree or disagree? Explain your reasoning.

SOCIAL STUDIES
Sales tax is collected by the local or state government to help pay for services to people who live in the city or state that collects the tax.
LESSON 21-3 PRACTICE

5. An entertainment news reporter stated that “about \(33\frac{1}{3}\%\) of Americans love listening to deejays, which is about 106,000,000 people.” At the time the reporter made that statement, about how many people were in the United States?

6. 62\% of Nate’s class came to see one of his performances. If 186 students saw his performance, how many students are in Nate’s class?

7. Isaac and Nate made enough money to pay off their startup loan and go shopping. Nate wants to buy a pair of basketball shoes that are on sale for 35\% off. If Nate paid $70, what was the original cost of the shoes?

8. A compact MP3 player costs $52 after 4.5\% sales tax. What was the original price?

9. Nate ordered a pizza to be delivered. The bill with 5\% tax and 20\% tip was $24.00. What was the original cost of the pizza?

10. Isaac’s mother gave him a subscription to an entertainment magazine for his birthday. The magazine was offered at 56\% off of the cover price. She paid $1.98 an issue. What was the cover price of the magazine?

11. **Reason abstractly.** Explain why the solutions to \(\frac{12}{w} = \frac{25}{100}\) and \(12 = 25\% \times w\) are the same.
ACTIVITY 21 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 21-1

1. 54 out of 72 of Isaac’s songs in a CD collection are pop songs. What percent of this CD collection are pop songs?

2. Write each amount as a percent.
   a. 23 songs out of 92
   b. $8 of $24 earned as profit

3. Nate tells his mom about a great deal he found on MP3 players. “They were originally $110.60 but now they are on sale for $88.48!” Use percents to determine whether this is a good deal. Explain your answer.

4. At the end of one performance, the total that Isaac received was $750. This included a tip for the usual fee of $649. What percent was the tip?

5. Nate’s savings account for the business had $12.10 more at the end of the year than the $252 it had at the beginning of the year. What percent more was in the savings account?
   A. 2.4%  B. 4.6%
   C. 4.8%  D. 48%

6. A subwoofer box for sound costs $260.40 after a price increase. The cost before the price increase was $240.00. What was the approximate percent of the price increase?
   A. 7.8%  B. 8.3%
   C. 8.5%  D. 9%

Lesson 21-2

7. The wholesale price of a special speaker was $64. It was then marked up 85% and sold online. What was the online price?

8. Isaac gave the receipt below to a customer as a bill. The customer was asked to fill in the blanks. What was the amount of the total bill?

   THANK YOU!
   IN Deejay Service
   Fee = $850.00
   20% Tip = $____
   Total Bill = $____
   A. $170  B. $680
   C. $708.33  D. $1,020

9. The extra audio parts that would have cost $118 at a supply store increased by 6.75%. What was the new cost of these parts?

10. Which of the pairs of values will give the equivalent final sale price?
   A. a discount of 20% off of $80
   B. a sale of \(\frac{1}{4}\) off the original price of $100
   C. a markup of 150% on $30

11. Isaac’s father earned 5.2% interest on his investments last year. If he had $40,000 invested, what was the balance in his account at the end of the year?
   A. $2,080  B. $42,000
   C. $42,080  D. $63,840
12. The sales tax on a $120 bill is 7.25%. After a coupon discount of 10% off the total cost with tax, what was the final amount of the bill?
   A. $99.30  B. $115.03  C. $115.83  D. $128.70

13. Complete the table by finding the percent of each number. Describe the relationship you see among the values in each column.

<table>
<thead>
<tr>
<th></th>
<th>10%</th>
<th>25%</th>
<th>$33\frac{1}{3}$%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$120$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$240$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$360$</td>
<td></td>
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</tr>
</tbody>
</table>

Lesson 21-3

14. How much money must Nate deposit in a savings account that pays 4% simple annual interest to earn $50 the first year?

15. The interest on several accounts is shown below. Each interest rate is simple annual interest. Which account balance was the highest at the beginning of the year?
   A. $42 interest earned at 2.8%
   B. $48 interest earned at 2.5%
   C. $50 interest earned at 2.2%
   D. $45 interest earned at 3.1%

16. Nate hears that $33\frac{1}{3}$% of the teen clubs in his area, or 6 clubs, offer discount tickets to students. Write an equation that could be used to determine the total number of teen clubs in the area.

17. Nate estimates that a search for websites about music produces 500,000 websites. If this is a 12% increase over the previous year, explain how you would find the number of websites for the previous year. Then find the approximate number of websites the previous year.

18. Nate knew that most mobile phones also included music players. In 2012, the number of mobile phones in Italy was 147.4% of the population. If the number of mobile phones was 88,600,000 in Italy in 2012, what was the approximate population?

19. Nate is selling a DVD of one performance for $19.99. This is the price after a discount of 25%. What was the original price of the DVD?
   A. $14.99  B. $25.00  C. $26.65  D. $79.96

MATHEMATICAL PRACTICES
Reason Quantitatively

20. People often leave tips between 10% and 25% at the teen clubs where the boys deejay, depending on how well they like the service and music. What was the price range of the original bill if a tip of $22 was given?
Write your answers on notebook paper. Show your work.

Every month the Incredible Ice Cream Shop sends a card with a coupon for a free ice cream treat to the members of their ice cream club who were born in that month. The number of members and their birthday month is shown below.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Club Members Born in the Month</th>
<th>Month</th>
<th>Number of Club Members Born in the Month</th>
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<tbody>
<tr>
<td>January</td>
<td>6</td>
<td>July</td>
<td>24</td>
</tr>
<tr>
<td>February</td>
<td>12</td>
<td>August</td>
<td>11</td>
</tr>
<tr>
<td>March</td>
<td>18</td>
<td>September</td>
<td>18</td>
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<tr>
<td>April</td>
<td>14</td>
<td>October</td>
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</tr>
<tr>
<td>May</td>
<td>10</td>
<td>November</td>
<td>27</td>
</tr>
<tr>
<td>June</td>
<td>13</td>
<td>December</td>
<td>33</td>
</tr>
</tbody>
</table>

1. What percent of the total number of coupons are sent out in October through December? Explain your reasoning.

2. Give the fraction, decimal, and percent that represent the number of club member birthdays in January, February, March, April, and June in the table below. Round to the nearest whole percent.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
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<tbody>
<tr>
<td>January</td>
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<tr>
<td>June</td>
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</tbody>
</table>

3. The manager of the ice cream shop noticed that 90% of all club members brought along a family member who spent an average of $18 in the shop. The tax on their bill was 5.5%.

   a. What was the total amount spent by the family members in one year?
   b. What would each family member have to spend, on average, to generate $5,000 in income before sales tax?
   c. What would each family member’s average bill be with sales tax?
## Scoring Guide

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 2, 3a-c)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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</thead>
<tbody>
<tr>
<td>Effective understanding and accuracy in calculating percents and finding a part given a percent.</td>
<td>Few if any errors in calculating percents and finding a part given a percent.</td>
<td>Multiple errors in calculating percents and finding a part given a percent.</td>
<td>Incorrect or incomplete understanding of calculating percents and finding a part given a percent.</td>
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<table>
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<tr>
<th>Problem Solving (Items 3a-c)</th>
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<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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<tbody>
<tr>
<td>An appropriate and efficient strategy that results in a correct answer.</td>
<td>A strategy that may include unnecessary steps but results in a correct answer.</td>
<td>A strategy that results in some incorrect answers.</td>
<td>No clear strategy when solving problems.</td>
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<tr>
<td>Accuracy in interpreting a percent to solve a problem.</td>
<td>Interpretation of a percent to solve a problem.</td>
<td>Difficulty interpreting a percent to solve a problem.</td>
<td>Incorrect interpretation of a percent to solve a problem.</td>
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<th>Mathematical Modeling / Representations (Item 2)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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<tr>
<td>Clear and accurate understanding of writing a ratio as a fraction, a decimal, and as a percent.</td>
<td>Writing a ratio as a fraction, a decimal, and as a percent with few or no errors.</td>
<td>Difficulty writing a ratio as a fraction, a decimal, and as a percent.</td>
<td>Little or no understanding of writing a ratio as a fraction, a decimal, and as a percent.</td>
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<th>Reasoning and Communication (Item 1)</th>
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<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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<tbody>
<tr>
<td>Precise use of math terms and language to explain calculating a percent.</td>
<td>An adequate explanation of calculating a percent.</td>
<td>A misleading or confusing explanation of calculating a percent.</td>
<td>An incomplete or inaccurate explanation of calculating a percent.</td>
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